1.8. Correlation function

Correlation and autocorrelation functions are still actively used in science, including in economics. If there are two time-varying random variables x_t and y_t, then they can be interconnected with each other not only at this timepoint t. It is quite possible that this relationship has some lag in time and the variable y_t depends not on x_t, but on the same variable that was seen on τ observations earlier, that is, - y_t depends on $x_{t-\tau}$.

Most often, the lag value τ *is* unknown to the researcher and should be found. This, in particular, is what the correlation function is used for.

If we calculate the pair correlation coefficients r between y_t and x_t, after that between y_t and x_{t-1}, between y_t and x_{t-2}, etc., then the change in the pair correlation coefficient depending on the lag will be a correlation function in tabular discrete form.

If the researcher studies the correlation not between two variables, but studies the influence of the previous values of the variable on its current values, then he calculates the coefficients of pair correlation between y_t and yt_{-1}, between y_t and y_{t-2}, between y_t and y_{t-3}, etc. The change in the pair correlation coefficient from the time shift of the series is called an autocorrelation function.

It is considered that the lags between variables or autoregression lags are detected by such a backward shift τ, at which the modulus of the pair correlation coefficient modulo is not less than *0,8*. Since the pair correlation coefficient can be both positive and negative, it is argued that with a positive coefficient value, the past value of the indicator corresponding to this lag has a positive effect on the current value. In the case when the pair correlation coefficient for some lag is negative, it means the reverse effect of past values with such a lag on the current value of the variable.

We will not analyze the degree of correlation and autocorrelation functions suitability in economic forecasting tasks. We will draw attention to the fact that in the case of using complex economic variables, it also becomes possible to construct correlation and autocorrelation functions, but since the complex coefficient of paired correlation consists of real and imaginary parts, it is necessary to consider complex-valued correlation and autocorrelation functions.

The peculiarity of the complex-valued correlation (autocorrelation) function is that this function has real and imaginary parts. And each of them will represent a dependence on the time shift:

$$\mathrm{Re}(r_{cXY}) = f_r(\tau); \quad \mathrm{Im}(r_{cXY}) = f_i(\tau) . \quad (1.8.1)$$

It is convenient to study the correlation function by depicting it graphically - when time shifts are plotted along the abscissa axis, and the corresponding values of the pair correlation coefficients are plotted along the ordinate axis. Such a graphical model is called a "correlogram" or "autocorrelogram", if an autocorrelation function is considered. Fig. 1.6 shows an example graph of such a correlogram.

With regard to the complex correlation function, several options for constructing a correlogram are possible.

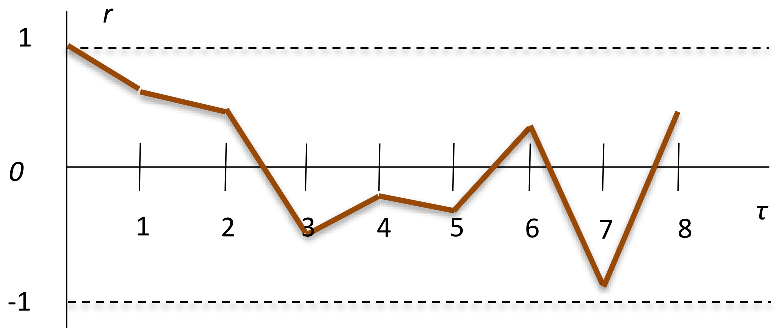

Figure 1.6. An example of a correlogram for real variables

The first option is a simple development of the correlogram of real variables, when the graphs of the real and imaginary parts are located on the same graph. In this case, a time shift is plotted along the abscissa axis, and the real Re(r_{cXY}) and imaginary Im(r_{cXY}) parts of the complex coefficient of pair correlation are located on this graph depending on the values they take for each lag.

In order not to invent the form of such a "parallel" correlogram, two series (2830 and 2831) from the database of the International Institute for Forecasters (*Makridakis, 2000*) were used. These two series of 103 observations each, were combined into a single complex variable. Then, for the resulting series of this complex variable, a complex autocorrelation function was calculated. The results of the calculations are shown in Table 1.2.

Table 1.2.

Example of an autocorrelation function for a complex series composed of series 2830 and 2831

Lag, τ	Re(r_{cXY})	Im(r_{cXY})
1	-0,757	0,178
2	-0,554	0,286
3	-0,480	0,267

4	-0,415	0,277
5	-0,377	0,274
6	-0,356	0,257
7	-0,345	0,242
8	-0,335	0,198
9	-0,332	0,196
10	-0,319	0,195
11	-0,288	0,206
12	-0,222	0,193
13	-0,180	0,201
14	-0,139	0,212
15	-0,142	0,193
16	-0,199	0,148
17	-0,251	0,120
18	-0,260	0,120
19	-0,269	0,114
20	-0,253	0,131
21	-0,210	0,160

Using the data in Table 1.2, it is possible to depict a complex autocorrelogram in such a parallel form (Fig. 1.7).

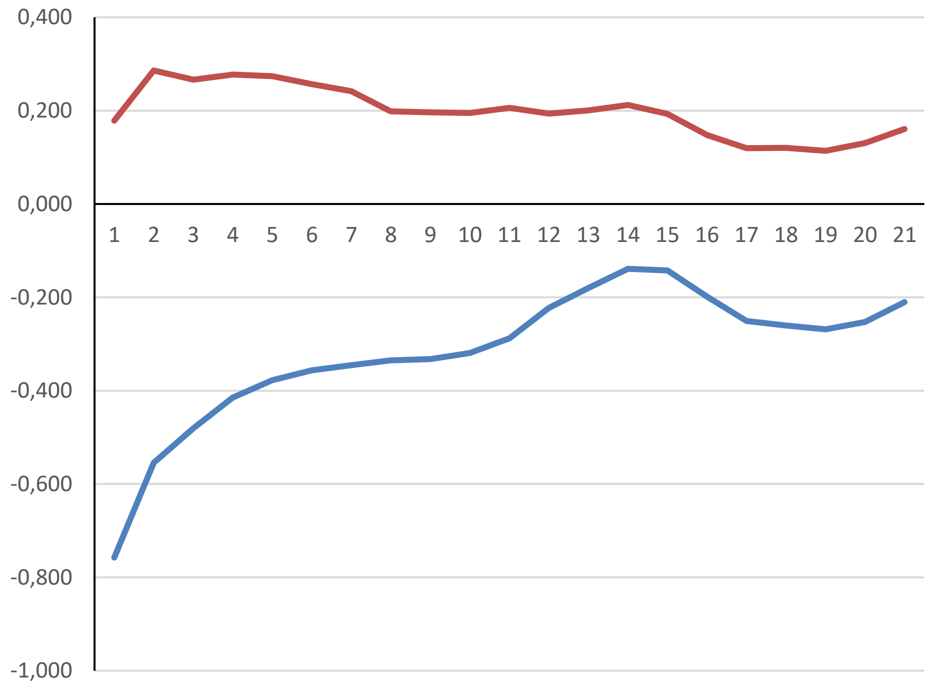

Figure 1.7. The first type of autocorrelogram table 1.2

The autocorrelogram in Figure 1.7 is similar to the autocorrelograms of real variables and its interpretation does not cause any particular difficulties. Here, the imaginary part of the complex correlation coefficient

Im(r_{cXY}) is shown in red – it is positive over the entire calculated interval; therefore, it is located above zero, and the blue color shows the correlogram of the real part Re(r_{cXY}). It is located below zero, because all of its values happened to be negative.

It is known from the previous paragraph that the real part of the complex coefficient of pair correlation characterizes the approximation degree of the complex-valued dependence to a linear form, and the imaginary part reflects the degree of dispersion around this line. It follows from the analysis of the given complex autocorrelogram, that the series under consideration can be described by the first-order autoregression, but good accuracy cannot be expected from this model, since the real part is less than 0.8, and the imaginary part is far from zero.

Another form of a complex correlogram is also possible. Since each lag value corresponds to one value of the complex coefficient of pair correlation, consisting of the real and imaginary parts, the correlogram can be considered as the trajectory of a point on the phase plane, and the correlogram itself as a phase portrait of the structure of the series.

Fig.1.8 shows a correlogram on such a phase plane, the axes of which are the real and imaginary parts of the complex correlation coefficient (in this case, autocorrelation).

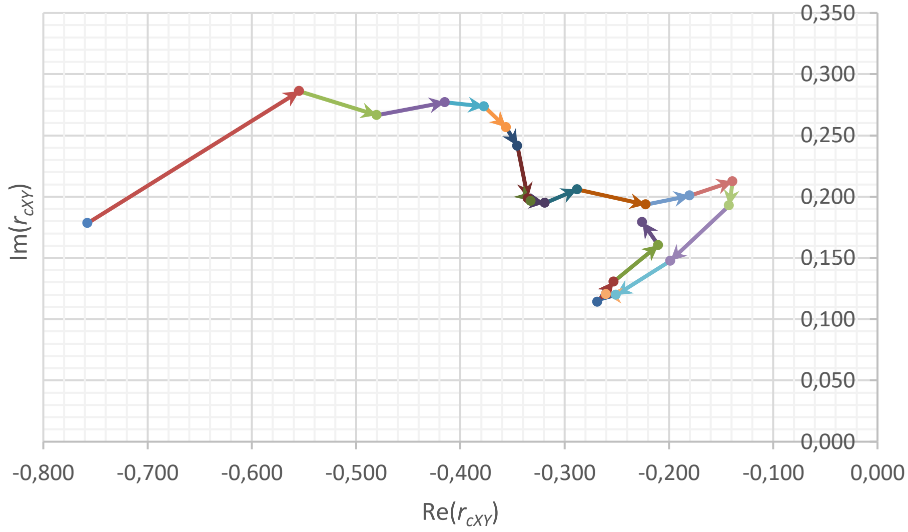

Figure 1.8. Autocorrelogram of Table 1.2 as a phase portrait

Here, the points on the complex plane of the correlogram are connected to each other not just by straight line segments, but by directed segments (arrows), the direction being maintained in strict order - from the first value of the complex coefficient of pair correlation to the last one, as it has been done in the figure.

This is a different autocorrelogram representation, the one which presents the process in a way different from the one in Figure 1.7. Here it is possible to obtain a variety of figures, the interpretation of which being a separate task. Phase portraits of complex autocorrelograms can have a very different shape and be located in different quadrants of the phase plane. For example, you can get a closed figure like a hysteresis loop, which will correspond to the cycle length if the process develops cyclically, for example, as shown in Fig.1.9.

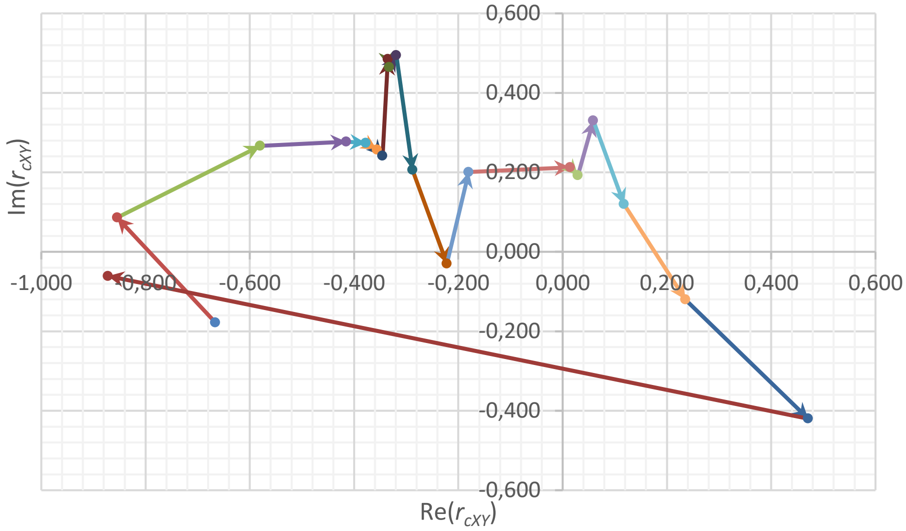

Figure 1.9. Possible phase portrait of the correlogram

If we consider a parallel complex correlogram, then the presence of such a loop may not be noticed, as it follows from Fig. 1.10. It shows the same complex correlogram as in the phase plane in Fig. 1.9.

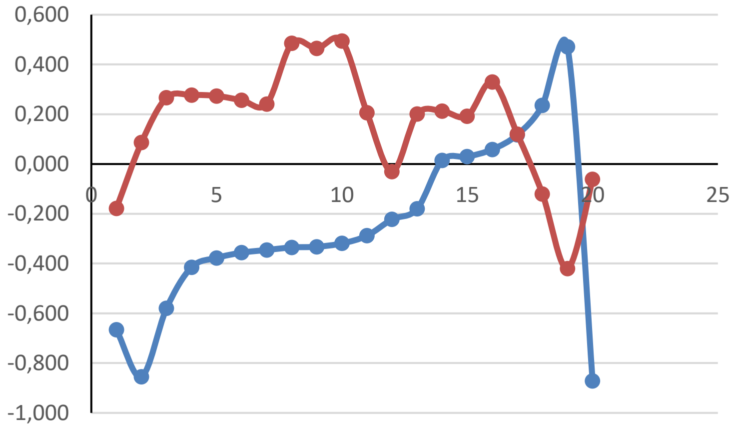

Figure 1.10. The complex correlogram corresponding to the case in Fig. 9, but presented in a parallel form.

Since the parallel form of the complex correlogram and its phase portrait give emphasis on different features of the correlation function, these two forms should be used in the analysis of the complex random series under study.