1.2. Complex -valued economy

Any complex number can be considered as a two-dimensional vector. And from (1.1.10) it follows that any complex-valued function can be considered as a function of a two-dimensional vector. Then it is clear that complex variables can be used where two economic indicators reflect different properties of one phenomenon or where the behavior of an object is determined by two factors

That is, where there is a dependence of one two-dimensional vector of economic indicators on another two-dimensional vector of economic indicators (or several two-dimensional indicators), it is very easy to describe such a dependence using models and methods of the complex variable function theory.

Where do such dependencies occur in the economy? Yes, almost at every step, because so many economic indicators are aggregated values that consist of several and, most often, two main values.

For example, gross output *Q* is made up of gross costs *C* and gross profit *G*, which makes it possible to represent the results as a complex variable *(C+iG).*

The capital that is used in production is also an aggregated value – it can be production *Ko* and non-production *K1*. Therefore, it can also be represented as a complex variable *(Ko+iK1).*

The labor used in production is also a generalized quantity. And it is also possible to take into account different effects of different f labor types on the results of production using a complex variable *(Lo+iL1).* Here *Lo* is the labor of production personnel, and *L1* is the labor of non-production personnel.

On stock exchanges, analysts monitor the dynamics of different companies` sales shares. In doing so, they use the aggregated value of transactions volume in shares:

$$Q_t = p_t q_t \,. \quad (1.2.1)$$

They will receive additional information if they use a complex variable *(pt+iqt),* having previously reduced the price per unit of *pt* shares and the volume of sales of *qt* shares at this price to a single scale and dimension. This possibility is demonstrated in the monograph "Complex-valued ..." (Svetunkov, 2012).

For the economy as a whole, at the meso or macro level, the volume of production can be divided into goods *P* and services *S* – and again, a complex variable *(P+ iS)* can be used for modeling.

In retail, these can be durable goods *Lo* and non-durable goods *Sh*, and again the sales volumes can be described by using a complex variable *(Lo + iSh).*

Similar examples can be continued further, but in any case, it is already clear that the scope of complex variables application in economics is extensive.

In this case, most often it does not matter which variable is attributed to the real part of the complex variable, and which to the imaginary one, because when forming a vector, such a problem is not worth it.

But when modeling the economy, there are such situations when the order of the variables, when they are formed into a complex number, takes on a certain meaning This may be the case, for example, when modeling the dependence of production results on production resources, that is, when constructing production functions.

In the domain of real variables, production functions most often take the form of power models of the type:

$$Q = aK^{\alpha} L^{\beta} . \quad (1.2.2)$$

And their difference from each other lies in the way of setting restrictions on the limits of change in degree indicators α and *β*.

Complex-valued analogues of this function can be very diverse, but the form of a complex-valued production function with real coefficients is universal (Svetunkov S.G., Svetunkov I.S., 2008):

$$C + iG = a(L + iK)^{\alpha} . \quad (1.2.3)$$

Here C stands for gross production costs,

G - is gross profit,

L - is labor costs,

K - capital.

This function can be represented as a system of two real equations:

$$\begin{cases} C = \mathrm{Re}[a(L+iK)^{\alpha}] \\ G = \mathrm{Im}[a(L+iK)^{\alpha}] \end{cases} \quad \text{or} \quad \begin{cases} C = a(\sqrt{L^2+K^2})^{\alpha} \cos(\alpha(arctg\dfrac{K}{L})) \\ G = a(\sqrt{L^2+K^2})^{\alpha} \sin(\alpha(arctg\dfrac{K}{L})) \end{cases} \quad (1.2.4)$$

It is obvious that in the range of real numbers, production functions of this kind do not occur.

It can be seen from (1.2.4) that it is precisely this order of assigning real numbers to the real or to the imaginary parts of complex variables that turns out to be important.

First of all, it should be noted that the real and imaginary parts of the complex production result of the model (1.2.3) react differently to the increase in each of the resources. So, with an increase in capital expenditures *K* with a constant expenditure of labor resources *L*, the cosine of the resources angle decreases, and the sine of this angle increases. Then it follows from the first equation of the system (1.2.4) that with the growth of capital, gross production costs decrease, and from the second equation of the system (1.2.4) it follows that gross profit increases. And that is exactly what happens in the real economy. That is, the complex-valued model (1.2.3) describes the details of the production process more accurately than the real model (1.2.2).

But here we have several more properties of the production function that are important from an economic point of view. If we present the complex production result in the exponential form of the record, then we get:

$$C + iG = \sqrt{C^2 + G^2} e^{iarctg\frac{G}{C}}. \quad (1.2.5)$$

What characterizes the polar angle of a complex production result? It characterizes the profitability of production (the ratio of gross profit to gross costs). That is, it is this order of attribution of gross profit to the imaginary part, and gross costs to the material part that makes economic sense. And since the polar angle of complex production resources characterizes the capital-labor ratio (the ratio of capital to labor), then the complex-valued production function (1.2.3) models the impact of capital-labor ratio on the production profitability. This testifies in favor of the fact that the model (1.2.3) has a bright economic sense.

But that is not all! Since the economic meaning of such equality is obvious

$$Q = C + G, \quad (1.2.6)$$

it turns out that the complex-valued model (1.2.3) describes not only the impact of resources on gross costs C and gross profit G, but also on gross output Q.

So, in some cases, the question of which part to attribute a particular economic indicator- to the real or to the imaginary part of a complex number - is important.

In order to use the apparatus of complex variables functions theory in economics when combining two economic indicators into one complex variable, the following conditions must be met, determined by the features of complex numbers:

1. These indicators should be two characteristics of one and the same process or phenomenon, that is, they should reflect different sides of this phenomenon;

2. They should also have the same dimension. In addition, they should have the same scale.

The first condition results from such reasons.

As a result of a complex variable formation from the two real variables, the complex variable is further considered as an independent single variable. Figuratively speaking, it carries information about its two constituent quantities and reflects the influence of each of these components on a certain result. These values must reflect different sides of the same phenomenon, otherwise their combination into one variable loses every meaning. They may be in close functional dependence with each other, or they may have a complex indirect relationship, but the main condition is that they must carry information about some common process for them. This is due to the fact that such characteristics of a complex number as its modulus and argument make sense only when the complex number components reflect the general content.

The second condition, which requires the same dimension of the complex variable components, is determined by the peculiarity of the complex number properties

Indeed, how can the modulus of a complex number be calculated if the real and imaginary parts have different dimensions, for example, rubles and pieces? There is no way to square each of them and add them – rub^2 cannot be added to pcs^2. Similarly, when calculating the polar angle, it is necessary to find the ratio of the imaginary part to the real part, and then find the arctangent of the resulting number. If the real and imaginary parts are of different dimensions, then nothing can be done, because the tangent of the angle is a dimensionless value, it cannot be measured in rubles / pieces.

In economics, a significant part of the indicators can be reduced to monetary units of measurement, for example, labor costs can be determined not in "man-hours", but in the cost of labor remuneration – by the amount of the wages fund at the enterprise or the enterprise subdivision. Therefore, the condition of the same dimension and scale in most of the real economic problems is quite feasible. But in the case when it is impossible to do this, each of the indicators should be reduced to relative dimensionless values in the way that turns out to be the best for the chosen form of the model.

Hereinafter we will assume that all these conditions are met and we can work with economic random complex variables